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Bulk Superconductor in Weakly Static Electric Field

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We investigate the properties of bulk superconductors in weakly static electric fields by a stationary perturbation method applied to the Schrödinger equation.

1. INTRODUCTION

Studies have long been made of the properties of superconductors in a weak static electric field. Weakly connecting superconductors (Fairbank *et al.*, 1988; Tinkham, 1975), beginning from the excellent experiment of Giaever (1974) and the naive prediction of Josephson (1974), have been studied since 1958. However, the above works are based upon the quantum tunneling effect, and just this limits the size of the superconductors greatly. Examples include the tunneling junction (Matisoo, 1972), the superconducting bridge (Gregers–Hansen, 1972), the point connection (Zimmerman, 1972), and the SNS junction (Clarke, 1969).

It is known that the quantum tunneling effect is not suited to a bulk superconductor. The starting point of our study of bulk superconductors is the cubic Schrödinger equation (Rogers and Shadwick, 1982; Whitham, 1974) in the electromagnetic field:

$$i\hbar \frac{\partial}{\partial t}\Psi = \left[\frac{1}{2\mu}\left(\frac{\hbar}{i}\nabla - \frac{q}{c}\mathbf{A}\right)\left(\frac{\hbar}{i}\nabla - \frac{q}{c}\mathbf{A}\right) + q\Phi\right]\psi + \beta|\Psi|^2\psi \qquad (\beta > 0) \quad (1)$$

In (1), we assume that all electrons in the bulk superconductor have been paired, i.e., the superconductor is in Feynman's (1966) ideal case.

Let

$$\psi(\mathbf{r}, t) = [\rho(r, t)]^{1/2} e^{i\theta(r,t)} \qquad (2)$$

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Inserting (2) into (1), we have

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \tag{3a}$$

$$\hbar \frac{\partial \theta}{\partial t} = -\frac{\mu}{2} v^2 - q\phi + \frac{\hbar^2}{2\mu} \frac{1}{\sqrt{\rho}} \nabla^2 \sqrt{\rho} + \beta \rho \sqrt{\rho} \tag{3b}$$

and

$$\mu \mathbf{v} = \hbar \nabla \theta - \frac{q}{2} \mathbf{A}, \qquad \text{where} \quad q = -2e, \quad \mu = 2m_e \tag{4}$$

Analyzing (3a), (3b), we have the following comments:

A. In the interior part of a bulk superconductor, i.e., when we consider an area far enough from the surface, the bulk superconductor can be regarded as approximately homogeneous, and the third term of (3b) on the right-hand side equals zero; i.e., $\nabla^2 \sqrt{\rho} = \nabla^2 \psi_0 = 0$.

B. The complement of A is that we do not explore the boundary between two superconductors, nor some alloys.

C. $|\psi|$ can be omitted when $T < T_c$ in the first stage of nucleation of the superconducting phase (near H_{c2}).

So, the fourth term of (3b) on the right-hand is equal to zero.

Then (3b) can be written as

$$\hbar \frac{\partial \theta}{\partial t} = -\frac{\mu}{2} v^2 - q\phi \tag{5a}$$

The meaning of (5a) is that the binding energy of Cooper pairs is affected by the exterior static electric potential. In other words, when $H \sim H_{c2}$, (5a) turns into

$$\frac{1}{2} \hbar \omega_c = -q\phi \qquad (\text{taking } n = 0)$$

$$\omega_c = \frac{2eH}{\mu c} \tag{5b}$$

In (5b) $\hbar\, \partial\theta/\partial t = (n + \frac{1}{2})\hbar\omega_c$ and $n = 0, 1, 2, \ldots$; $v = v_z = 0$ (de Gennes, 1966). We can get the following relationship from (5b):

$$\phi_c = \frac{\hbar H_{c2}}{2\mu c} \tag{6}$$

ϕ_c is called the nucleating electric potential of the superconducting phase, and $\phi_c \leq 10^{-10} H_{c2}$.

2. BULK SUPERCONDUCTORS IN WEAK ELECTRIC FIELD

Because $\phi_c \leq 10^{-10} H_{c2}$, the measurement of H_{c2} is easier. It seems likely that the discussion of ϕ_c has only theoretical interest. However, we may investigate the properties of a bulk superconductor in the exterior static electric field $\mathscr{E}$, not an exterior electric potential ϕ.

For supercurrent $\mathbf{j}_s$ proportional to $|\Psi|^2$, $\mathbf{j}_s$ then equals zero and the static electric field $\mathscr{E}$ and the static magnetic field $\mathbf{H}$ can be regarded as homogeneous.

Let $\mathbf{H}$ propagate along the Z axis and $\mathscr{E}$ propagate along the X axis. We have (Fairbank *et al.*, 1988; Tinkham, 1975; de Gennes, 1966)

$$\hat{H}\Phi(x) = E\Phi(x) \tag{7}$$

where

$$\hat{H} = \hat{H}_0 + \hat{H}', \qquad E = E^{(0)} + \Delta E \tag{8}$$

$$\hat{H}_0 = -\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + \frac{1}{2}\mu\omega_c^2(x - x_0)^2 \tag{8a}$$

$$\hat{H}' = -q\mathscr{E}x, \qquad \omega_c = \frac{2eH}{\mu c} \tag{8b}$$

$$E^{(0)} = |\alpha| - \frac{\hbar^2 k^2}{2\mu}, \qquad x_0 = \frac{\hbar k c}{2eH} \tag{8c}$$

Equation (7) can be treated by the singular perturbation method for the Schrödinger equation. When $n = k' = 0$, corresponding to (7), the lowest eigenvalue and eigenvector become

$$E_0^{(0)} = \frac{1}{2}\hbar\omega_c - \frac{q^2\mathscr{E}^2}{2\mu\omega_c^2} - q\mathscr{E}x_0 = E_0^{(0)} + \Delta E = |\alpha| + \Delta E \tag{9}$$

$$\Phi_0^{(1)}(x) = \Phi_0^{(0)}(x) + \sum_n{}' \frac{H'_{n0}}{E_0^{(0)} - E_n^{(0)}}\Phi_n^{(0)} \tag{10}$$

$$= \Phi_0^{(0)}(x) + \frac{q\mathscr{E}}{\hbar\omega_c}\xi 2^{-1/2}\Phi_1^{(0)}(x) \tag{11}$$

where (Fairbank *et al.*, 1988; Tinkham, 1975, de Gennes, 1966)

$$\xi^2(T) = \frac{\hbar^2}{2\mu|\alpha|}$$

$$\Phi_0^{(0)}(x) = K_0' \exp\left\{-\frac{1}{2}\frac{(x - x_0)^2}{\xi^2(T)}\right\} \tag{12}$$

$$\Phi_1^{(0)}(x) = K_1(x - x_0)\exp\left\{-\frac{1}{2}\frac{(x - x_0)^2}{\xi^2(I)}\right\}$$

Let us look at the physical meaning of (9) and (11):

A. In (9): For a bulk superconductor, all energy levels $E^{(0)}$ ($n = 0, 1, 2, \ldots$) will drop to $E_n^{(0)} - |\Delta E|$ upon taking into account the exterior electric field $\mathscr{E}$.

B. In (11): Taking into account $\mathscr{E}$, the lowest superconductive state vector $\Phi_0^{(1)}(x)$ is a mixed wave function of $\Phi_0^{(0)}(x)$ and $\Phi_1^{(0)}(x)$. The eigenvalue $E_1^{(0)}$ of the latter is the energy level neighboring $E_0^{(0)}$.

3. CONCLUSION AND DISCUSSION: THE CRITICAL VALUE OF $\mathscr{E}$

The reversal between normal and superconductive states of a bulk superconductor could be fine-tuned by changing $\mathscr{E}$. That is, from (9) we get

$$E_0^{(1)} = |\alpha| = |\Delta E| \tag{9a}$$

We call $\mathscr{E} = \mathscr{E}_c$, which preserves $E^{(1)}$ unchanged, the critical electric field strength. In the presence of $|\alpha|$, we can expect $\mathscr{E}_c$ to play an important part in the nucleating area near the surface of a bulk superconductor. In other words, we may be concerned with the relationship between $\mathscr{E}_c$ and H_{c3}. This will be discussed in another paper.

REFERENCES

Clarke, J. (1969). *Proceedings of the Royal Society A*, **308**, 447.

De Gennes, P. G. (1966). *Superconductivity of Metals and Alloys*, Benjamin.

Fairbank, J. D., Deaver, B. S., Everitt, C. W. F., Jr., Michelson, P. F., eds. (1988). *Near Zero: New Frontiers of Physics*, Freeman, New York.

Feynman, R. P. (1966). *The Feynman Lectures on Physics*, Vol. 3.

Giaever, I. (1974). *Les Prix Nobel en 1973*, Nobel Foundation.

Gregers-Hansen, P. E. (1972). In *Proceedings Applied Superconductivity Conference*, p. 597.

Josephson, B. D. (1974). In *Lex Prix Nobel en 1973*, Nobel Foundation.

Matisoo, J. (1972). In *Proceedings Applied Superconductivity Conference*, p. 535.

Rogers, C., and Shadwick, W. F. (1982). *Backlund Transformations and Their Applications*, Academic Press.

Tinkham, M. (1975). *Introduction to Superconductivity*, McGraw-Hill, New York.

Whitham, G. B. (1974). *Linear and Nonlinear Waves*, Wiley, New York.

Zimmerman, J. E. (1972). In *Proceedings Applied Superconductivity Conference*, p. 544.